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Elastic interaction graphs for steel H-sections subjected to bending, shear and axial forces

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Abstract

Interaction graphs are extensively used as a tool for designing prismatic member sections subjected to several combined stresses. For a long time, sets of these graphs have been available in building technologies: reinforced concrete, steel and composite sections, under various stresses.

This paper shows the general formulation for obtaining the graphs corresponding to bending, shear and axial forces of H-shaped steel sections. A general study of the stresses interaction limit surface is made, including a detailed description of its three regions, and the generation of interaction graphs corresponding to their sections by constant axial values. © 1999 Elsevier Science Ltd. All rights reserved.

1. Introduction

The indisputable convenience of using interaction diagrams of several combined efforts (generally up to three), on a prismatic member section, is the cause for the spread of those graphs. Among these can be mentioned, as being universal, those corresponding to reinforced concrete sections elaborated by Jiménez Montoya et al. (1991); we can also cite those available for metallic or mixed profiles in several publications, among which stand out the works by Atsuta and Chen (1976), Zhou and Chen (1985), Bradford (1991) and E.C.S. (1992).

The interaction between normal and tangential stresses expressing Mises' yield criterion, applied to the most stressed point in a metallic section, is usually considered as a limit for the elastic design of the resistant element.

Stresses expression, that depending on the efforts gives the Resistance of Materials, allows application of that limit condition directly to them. When this criterion form is given, the interaction limit surfaces for trios of applied efforts result in planes, quadrics, more complex surfaces, or a mixture of them.

In this paper, the obtaining of the interaction limit surface among axial, bending and shear forces for

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H-shaped metallic sections (series IPN, IPE, HEB, HEA, HEM) and the diagrams corresponding to their sections by planes of axial constant force are shown.

2. Problem outlining

In Fig. 1 the notation and the considered stresses in the H-shaped section are shown. Those stresses are shown in this way

$$\sigma = \frac{N}{A} + \frac{M}{I} y = v + \mu y$$

$$\tau = \frac{Q}{I}\frac{M_{\mathrm{est}}}{e} = \begin{cases} q \cdot a(s) & \text{in the flange} \\ q \cdot \left(b - \frac{1}{2}y^2\right) & \text{in the web} \end{cases}$$

being

$$v = \frac{N}{A}$$

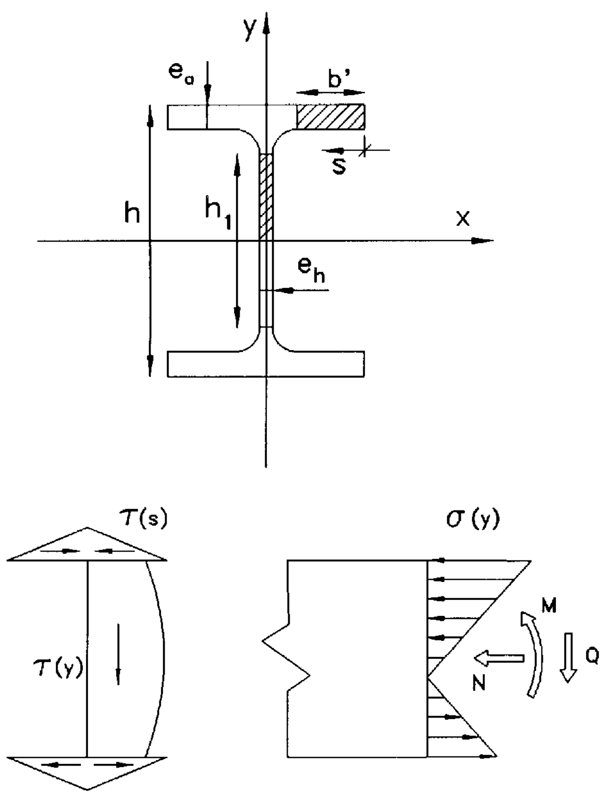

Fig. 1. Geometrics and stresses.

$$\mu = \frac{M}{I}$$

$$q = \frac{Q}{I}$$

$$a(s) = (h - e_a)\frac{s}{2}$$

$$b = \frac{S_x}{e_h}$$

$S_x =$ static first order moment of a half section, about x-axis.

$I =$ inertia moment of the section, about x-axis

$A =$ section area

With Mises' criterion, the square of comparison stress will be

$$\sigma_e^2 \geqslant \sigma_{co}^2 = \sigma^2 + 3\tau^2 = \begin{cases} (v + \mu y)^2 + 3q^2 a^2(s) \text{ in the flange} & (1) \\ (v + \mu y)^2 + 3q^2 \left(b - \dfrac{y^2}{2}\right)^2 \text{ in the web} & (2) \end{cases}$$

3. Searching of the most stressed point

3.1. In the flange

The most solicited point in the flange is defined by $s = b'$, $y = h/2$ (Fig. 1) in which an absolute maximum of σ_{co}^2 is produced; and when this one is the most stressed of the section, the limit yield condition will be:

$$\sigma_e^2 = v^2 + 2\frac{h}{2}v\mu - \frac{h^2}{4}\mu^2 + 3q^2 a_1^2 \tag{3}$$

being

$$a_1 = a(b') = (h - e_a) \cdot \frac{b'}{2}$$

3.2. In the web

The functions σ, τ, σ^2, τ^2 and $\sigma_{co}^2 = f(y)$ in the web are shown in Fig. 2, the last one being expressed as follows:

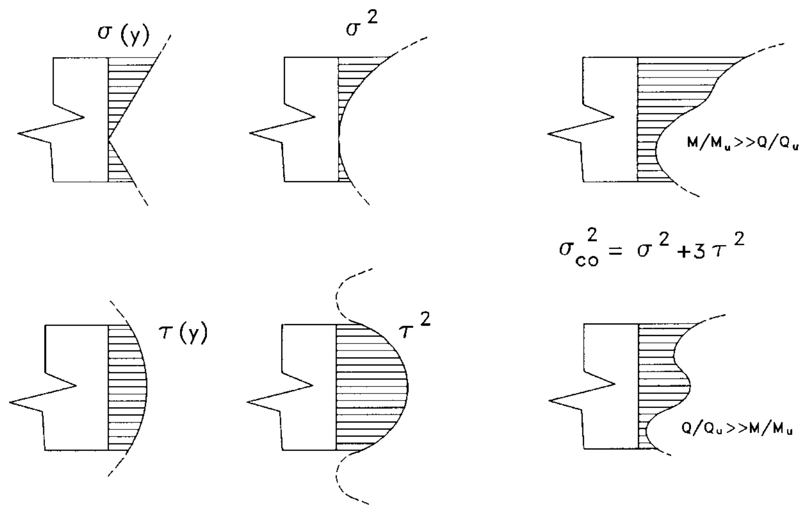

Fig. 2. Look of stress functions

$$f(y) = v^2 + 2v\mu y + \mu^2 y^2 + 3q^2\left(b^2 - by^2 + \frac{y^4}{4}\right)$$

Its highest value can be reached either in an absolute maximum at $y = h_1/2$, or in a relative one at whatever position $y \in [0, h_1/2]$ (the problem can be restricted to this interval, given the symmetry of τ^2) that we are going to study, vanishing its first derivative:

$$f'(y) = 2v\mu + 2\mu^2 y + 3q^2\left(y^3 - 2by\right) = 0 \tag{4}$$

Out of the plane $q = 0 = Q$, where the interaction diagram is, immediately, the straight line

$$\sigma_e = \frac{N}{A} + \frac{M}{W}$$
(W =Resistant module about x-axis)

the required condition is:

$$y^3 + \frac{2}{3q^2}\left(\mu^2 - 3bq^2\right)y + \frac{2v\mu}{3q^2} = 0 \tag{5}$$

a polynomial equation of third degree, which solutions are

$$y_1 = \frac{\sqrt{8\left(3bq^2 - \mu^2\right)}}{3q}\cos\frac{\theta}{3} \tag{6a}$$

$$y_2 = \frac{\sqrt{8\left(3bq^2 - \mu^2\right)}}{3q}\cos\left(\frac{\theta}{3} + \frac{2\pi}{3}\right) \tag{6b}$$

$$y_3 = \frac{\sqrt{8(3bq^2-\mu^2)}}{3q}\cos\left(\frac{\theta}{3}+\frac{4\pi}{3}\right) \tag{6c}$$

Being

$$\theta = \operatorname{arcos}\frac{-9v\mu q}{\sqrt{8(3bq^2-\mu^2)^3}} \in \left[\pi, \frac{3\pi}{2}\right] \tag{7}$$

In the plane $v = 0 = N$ the independent term of eqn (5) disappears and the solution reduces to:

$$y_1 = 0$$

$$y_{2,3} = \pm\sqrt{\frac{2(3bq^2-\mu^2)}{3q^2}}$$

Among the solutions of eqn (6) there is always a real one; the other two will be also real if this occurs simultaneously:

$$3bq^2 - \mu^2 > 0 \tag{8a}$$

$$\frac{9v\mu q}{\sqrt{8(3bq^2-\mu^2)^3}} < 1 \Longrightarrow 81v^2\mu^2q^2 - 8(3bq^2-\mu^2)^3 < 0 \tag{8b}$$

In fact, the performance of eqn (8a) is guaranteed with the performance of eqn (8b); therefore the condition is reduced to this last one.

The analysis of the second derivative of the function f:

$$f''(y) = 9q^2y^2 - 2(3bq^2-\mu^2) \tag{9}$$

shows that the condition (8a) is necessary for the existence of a relative maximum, but it is enough in the plane $v = 0$, where in $y_1 = 0$ a relative maximum appears and in y_2, y_3 relative minima appear.

In general, the existence of a maximum requires that

$$9q^2y_1^2 - 2(3bq^2-\mu^2) < 0 \tag{10}$$

Replacing y_1 from eqn (6a) remembering the trigonometric relation

$$\cos\theta = 4\cos^3\frac{\theta}{3} - 3\cos\frac{\theta}{3},$$

and after some algebraic manipulations, this expression becomes identical to eqn (8b); which is logical, given the form of the function f:

- If its first derivative vanishes only in a real point, it is a relative minimum as $f(y)$ is continuous and indefinitely derivable, and increasing for $y \to \pm\infty$.
- If it vanishes in three real points, one is maximum and the other two relative minima, by the same reason, as it can be proved replacing y_2 and y_3 in eqn (9). In this case, the relative maximum is always the intermediate solution, that is y_1 if in eqn (7) it is chosen

$$\theta \in \left[\pi, \frac{3\pi}{2}\right].$$

But, in fact, the existence of the relative maximum only will be the restricting condition when this one:

(a) Appears inside the web

$$y_1 < \frac{h_1}{2}$$

Replacing y_1 of eqn (6a), remembering the trigonometric expression aforementioned, and after some algebraic manipulations, this condition is expressed:

$$8h_1\mu^2 + 16v\mu + 3h_1q^2\left(h_1^2 - 8b\right) < 0 \tag{11}$$

(b) It surpasses the value of f at its end:

$$f(y_1) > f\left(\frac{h_1}{2}\right)$$

that is to say,

$$\mu^2\left(y_1 + \frac{h_1}{2}\right) + 2v\mu + 3q^2\left(y_1 + \frac{h_1}{2}\right)\left[\frac{1}{4}\left(y_1^2 + \frac{h_1^2}{4}\right) - b\right] < 0 \tag{12}$$

and

(c) It surpasses the value of σ_{co}^2 in the flange

$$f(y_1) > \sigma_{co}^2 \text{ (flange)}$$

that is

$$\mu^2\left(\frac{h^2}{4} - y_1^2\right) + 2v\mu\left(\frac{h}{2} - y_1\right) + 3q^2\left[a_1^2 - \left(b - \frac{y_1^2}{2}\right)^2\right] < 0 \tag{13}$$

The rationalization of eqns (12) and (13), taking into account the expression for y_1 is excessively long and takes to expressions of very high degree in q, μ, and v, that is why it will not be asserted.

3.2.1. When the absolute maximum is the value $f(h_1/2)$, the condition is expressed:

$$v^2 + 2v\mu\frac{h_1}{2} + \mu^2\,\frac{h_1^2}{4} + 3q^2a_2^2 - \sigma_e^2 = 0 \tag{14}$$

Being

$$a_2^2 = \left(b - \frac{h_1^2}{8}\right)^2$$

This one is to be retained only when this absolute maximum surpasses the corresponding value in the flange:

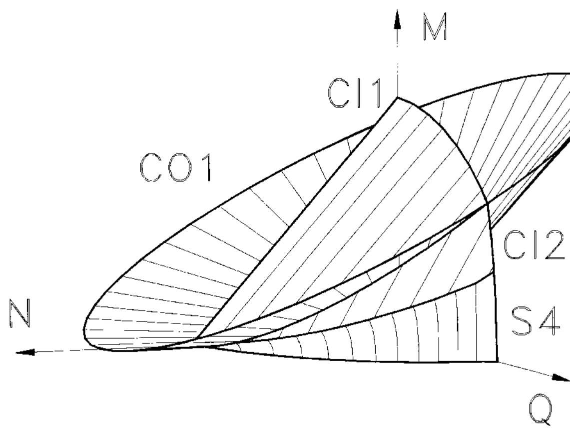

Fig. 3. Limit surfaces.

$$\mu^2\left(\frac{h^2}{4}-\frac{h_1^2}{4}\right)+2v\mu\left(\frac{h}{2}-\frac{h_1}{2}\right)-3q^2a_3^2<0 \tag{15}$$

Being

$$a_3^2=a_2^2-a_1^2>0.$$

3.2.2. Finally, when the maximum is relative in y_1, the yield condition will be:

$$(v+\mu y_1)^2+3q^2\left(b-\frac{y_1^2}{2}\right)^2-\sigma_e^2=0 \tag{16}$$

being equally impossible its rationalization in terms of q, μ, and v, bearing the expression of y_1 in mind.

4. Geometrical interpretation and complete discussion of the problem

The expressions (3), (14) and (16) are the interaction limit conditions for shear, bending and axial force, and in a space $q-\mu-v$ (or Q, M, N) they represent the equation of three surfaces. Each one of them is of application, as we have seen, in some fields of this space, delimited by the surfaces corresponding to the equations of the expressions (8b), (11), (12) (13) and (15) put in the limit of the equality to zero. Now we are going to study these surfaces.

4.1. Maximum in the flange

4.1.1. Field

The validity field of the surface, eqn (3), is the part of the space q-μ-v complementary to the given one by condition (15); in the limit:

$$\mu^2\left(\frac{h^2}{4}-\frac{h_1^2}{4}\right)+2v\mu\left(\frac{h}{2}-\frac{h_1}{2}\right)-3q^2a_3^2=0$$

The study of the quadratic surface (see Garcıa and López (1984) or any other Algebra Treatise) shows that it is a cone whose vertex is the origin, whose axis is contained in the odd quadrants of the plane $q = 0$ and whose elliptic guideline is tangent to the plane $\mu = 0$ along the axis (Ov) (Fig. 3). The field of application of eqn (3) is the inner part of cone that we name CO1.

4.1.2. Limit surface

The limit surface eqn (3), is another quadratic one; its study shows that it is a cylinder whose axis is also contained in the plane $q = 0$ and it passes over the origin, whose guideline is also elliptic (Fig. 3). We will refer to it as CI1.

4.2. Maximum in the web end

4.2.1. Field

The field of application of the limit surface, eqn (14), is given by eqn (15). That is to say, the region of the outer space of the cone CO1 (Fig. 3) but limited by the conditions (8b), (11), (12) and (13).

The condition (8b), in its limit, is the equation of a surface of upper order, that we will name S1, whose study reveals that:

- It is tangent to the plane $v = 0$ along its straight line $\mu = q\sqrt{3b}$.
- It is tangent to the plane $\mu = 0$ along the axis Ov.
- It is concave towards the axis μ+ in the first quadrant (Fig. 3).

The condition (11), in its limit, also shows a quadratic surface, a new cone that we will name CO2, with the same characteristics of CO1, and tangent to it along Ov.

The condition (12), in the limit, shows the equation of an upper order surface which we will name S2, with the same characteristics that S1, tangent to it along Ov, but it cuts the plane $v = 0$ in a straight line

$$\mu = q\sqrt{3b - 3\,\frac{h_1^2}{16}}, \qquad \text{of minor slope.}$$

The condition (13), in its limit, shows equally an upper order surface which we will name S3, similar to S1 and S2, that cuts the plane $v = 0$ in a straight line

$$\mu = \frac{2}{h}\,q\,\sqrt{3b^2 - 3a_1^2}.$$

Only for values of μ over any of the surfaces S1, S2 or S3, or inside the cone CO2 and simultaneously outside the cone CO1, the interaction limit surface, eqn (14), is of application.

4.2.2. Limit surface

This surface, eqn (14), is also a quadratic one. It is another elliptic cylinder which we will name CI2 (Fig. 3) with similar characteristics to CI1, whose axis passes over the origin but its slope is minor, and presents different eccentricities. The cone CO1, that delimits their validity fields, also contains the warped curve intersection of both cylinders.

4.3. Maximum inside the web

4.3.1. Field

The field of application of the limit surface third part is formed by the region lower to the surfaces S1, S2 and S3, and exterior to the cone CO2 simultaneously.

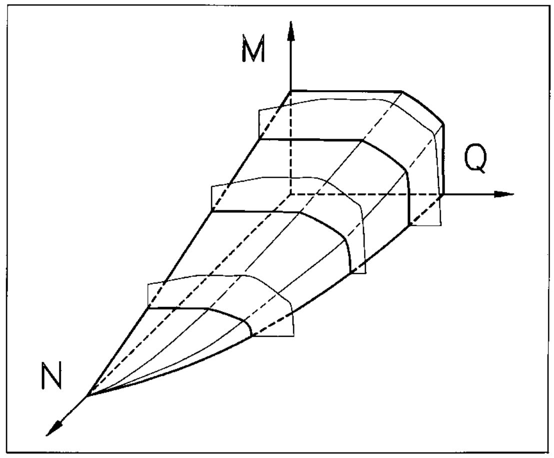

Fig. 4. Graphs obtaining by cutting limit surfaces.

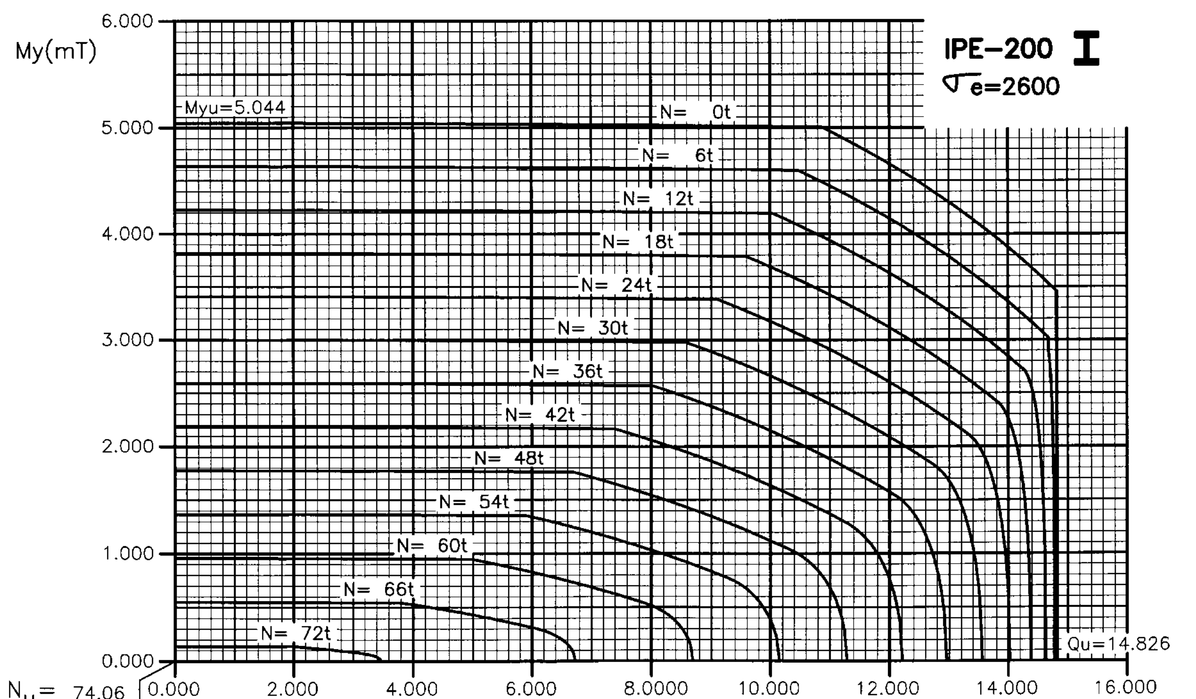

Fig. 5. An example for I-section.

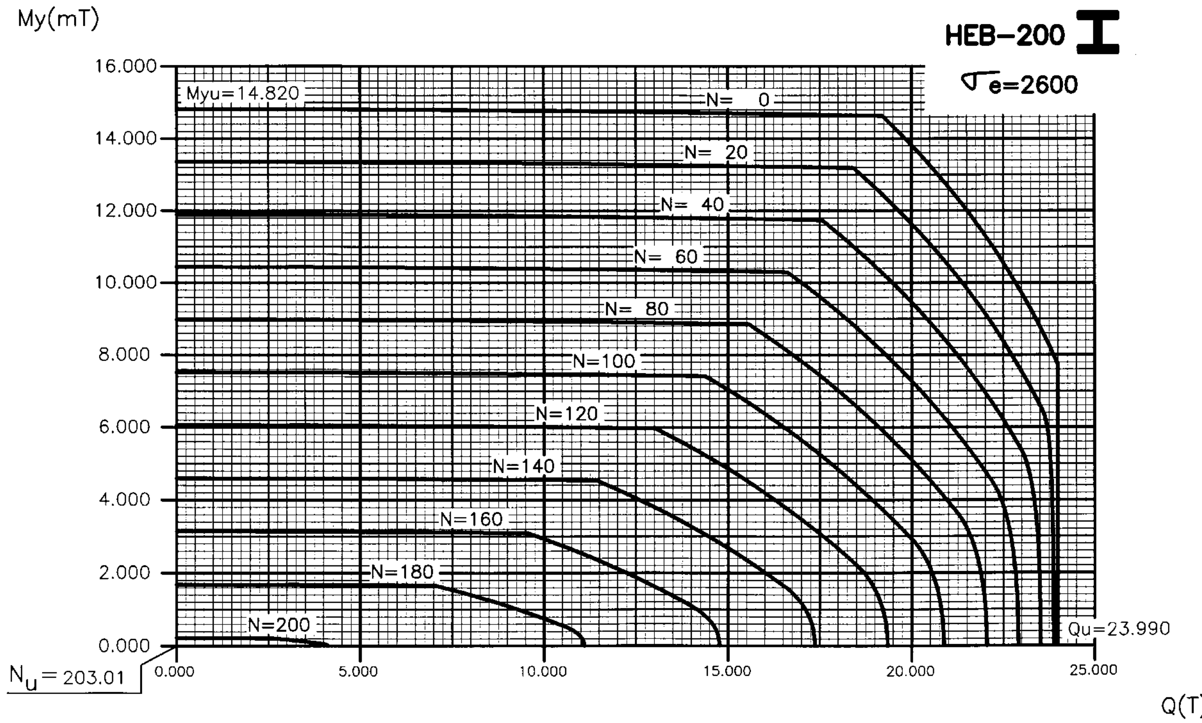

Fig. 6. An example for H-section.

4.3.2. Limit surface

The interaction surface, eqn (16), which we will name S4, is in this case of a very high order, and (Fig. 3):

Cuts the plane $v = 0$ along the straight line

$$q = q_u = \frac{\sigma_u e_h}{\sqrt{3} S_x}$$

Cuts the plane $\mu = 0$ along the ellipse

$$v^2 + 3q^2 b^2 = \sigma_e^2.$$

5. Limit surface representation—interaction diagrams

The limit surface is, then, formed by three portions of two elliptic cylinders and an upper order surface, secant between themselves. Given the validity fields configuration of each one of these portions, and their characteristics, the most suitable interaction diagrams are those produced by sections of the surfaces with constant axial force planes ($v = ct$); in this way, all the diagrams present a curve section of each surface, in its corresponding field, meanwhile the other two possibilities μ or q constant present different typology because not all the planes cut the three surfaces.

In Figure 4 the obtaining process of these graphs is explained. Figures 5 and 6 show the diagrams for IPE 200 and HEB 200, respectively, ready for use.

6. Conclusion

The analytical formulation for the obtaining of the limit surface of the axial force, major axis bending and concomitant shear force interaction, in metallic H-shaped sections, has been exposed.

This surface is formed by three regions, two of them quadratic and the third one is of upper order, their validity fields are equally delimited by quadratic and upper order surfaces in the space N–M–Q.

Once the surfaces have been obtained and studied, we have obtained the interaction diagrams corresponding to sections by constant axial force, which prove to be the most suitable, resulting in mixed lines of two elliptic parts and another of upper order, with discontinuity of derivative at the changing points.

Finally, as an example, the diagrams corresponding to an H-shaped section of narrow flange and another of wide flange from commercial series, have been represented.

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